Institute for Christian Teaching
Education Department of Seventh-day Adventist

GOD AND CALCULUS

by
Norie Grace Rivera-Poblete
Mission College
Muak Lek Saraburi, Thailand

450-00 Institute for Christian Teaching
12501 Old Columbia Pike
Silver Spring, MD 20904 USA

Prepared for the
27th International Faith and Learning Seminar
held at
Mission, Muak Lek Saraburi, Thailand
December 3 – 15, 2000

Introduction

Calculus is one of the greatest achievements of the human intellect. Sometimes called the " mathematics of changes", it is the branch of mathematics that deals with the precise way in which changes in one variable relate to changes in another. In our daily activities we encounter two types of variables: those that we can control directly and those that we cannot. Fortunately, those variables that we cannot control directly often respond in some way to those we can. For example, the acceleration of a car responds to the way in which we control the flow of gasoline to the engine; the inflation rate of an economy responds to the way in which the national government controls the money supply; and the level of antibiotics in a person's bloodstream responds to the dosage and timing of a doctor's prescription. By understanding quantitatively how the variables, which we cannot control directly, respond to those that we can, we can hope to make predictions about the behavior of our environment and gain some mastery over it. Calculus is one of the fundamental mathematical tools used for this purpose.

Calculus was invented to answer questions that could not be solved by using algebra or geometry. One branch of calculus, called Differential calculus, begins with a question about the speed of moving objects. For example, how fast does a stone fall two seconds after it has been dropped from a cliff? The other branch of calculus, Integral calculus, was invented to answer a very different kind of question: what is the area of a shape with curved sides? Although these branches began by solving different problems, their methods are the same, since they deal with the rate of change.

Some anticipations of calculus can be seen in Euclid and other classical writers, but most of the ideas appeared first in the seventeenth century. Sir Isaac Newton (1642 – 1727) and Gottfried W. Leibniz (1646 – 1716) independently discovered the fundamental theorem of calculus. After its start in the seventeenth century, calculus went for over a century without a proper axiomatic foundation. Newton wrote that calculus could be rigorously founded on the idea of limits, but he never presented his ideas in detail. A limit, roughly speaking, is the value approached by a function near a given point. During the eighteenth century many mathematicians based their work on limits, but their definition of limit was not clear. In 1784, Joseph Louis Lagrange (1736-1813) at the Berlin Academy proposed a prize for a successful axiomatic foundation for calculus. He and others were interested in being as certain of the internal consistency of calculus as they were about algebra and geometry. No one was able to successfully respond to the challenge. It remained for Augustin Louis Cauchy (1789-1857) to show, sometime around 1820, that the limits can be defined rigorously by means of inequalities (Hughes-Hallett, 1998, 78).

Purpose of the study

Calculus is one of the subjects being taught for higher mathematics in high schools and colleges. The purpose of this paper is to show how to use calculus in our relationship with God. I will employ parallelism and contrast to teach the values with the hope that through teaching calculus the teacher can bring his/her students closer to God.

Application

- God is the greatest mathematician. According to Avery J. Thompson, "Any credence given to the study of mathematics must recognize that God is the original mathematician. And though, through the ages, humankind has experimented to be able to draw conclusion in the areas of mathematics, God's laws are error-free and constant. His everlasting watch-care in the 'natural' cyclic phenomena of this earth daily proves His mathematical supremacy. Galileo is remembered for having acknowledged that 'mathematics is the language that God used to create the universe'".
- We are the variables and God is the constant. God doesn't change; He is the same God from the beginning. According to Malachi 3:6 (NIV) "I the Lord do not change." As variables, we depend on Him to give some predictability to life. Without some constancy, we would never be able to plan, or hope, or know what to expect. God's laws, both the moral law and the laws of nature, are as constant as He is. So we can expect that tomorrow the sun will rise in the east, as it has done every day in the past.
- A given value (constant) helps in solving a given function. For example, it is estimated that x months from now, the population of a certain community will be $P(x) = x^2 + 20x + 8{,}000$ (function). At what rate will the population be changing with respect to time 15 months (constant) from now? Solution: the rate

of change of the population with respect to time is the derivative of the population function. That is, $Rate of change = P'(x) = 2x + 20$. The rate of change of the population 15 months from now will be $P'(15) = 2(15) + 20 = 50$ people per month. God, who we said is constant, is a "present help in time of trouble"(Psalm 46:1). Indeed, without God in one's life, we will never find satisfactory solutions to problems.

- In calculus, if we violate the laws we will never find the right solution to the given problem or function. When we violate the laws of God our life become chaotic and we will never find peace or the right solutions to our problems.

Limit and Limitless God

The concept of limits is very important in calculus. Without limits calculus simply does not exist. Every single notion of calculus is a limit in one sense or another. On the contrary God has no limit. When we apply the concept of limit, we examine what happens to the y-values of a function ***f*(*x*)** as ***x*** gets closer and closer to (but does not reach) some particular number, called ***a***. If the y-values also get closer and closer to a single number, ***L***, then the number ***L*** is said to be the ***limit of the function as x approaches a***. Thus, we say that ***L is the limit of f(x) as x approaches a***. This is written in mathematical shorthand as $L = \lim_{x \to a} f(x)$

where the symbol → stands for the word *approaches*. If the y-values of the function do not get closer and closer to a single number as x gets closer and closer to ***a***, then the function has no limit as ***x*** approaches ***a***. Figure 1 shows the graph of a function that has a limit ***L*** as ***x*** approaches a particular ***a***.

Illustration:

$f(x) = x^2 - x + 1$ $\lim_{x \to 2} x^2 - x + 1 = 3$

X	1.0	1.5	1.9	1.95	1.99	1.995	1.999	2.0	2.001	2.005	2.01	2.05	2.1	2.5	3.0
f(x)	1.00	1.75	2.71	2.85	2.97	2.985	2.997		3.003	3.015	3.03	3.15	3.31	4.75	7.0

Left side → ← right side

Table 1

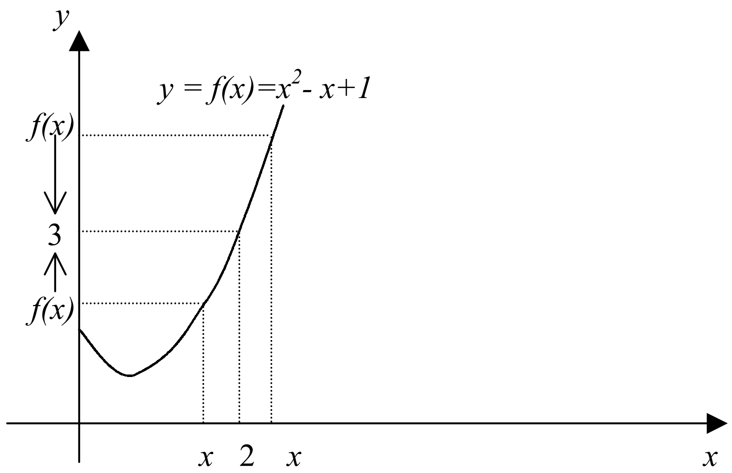


Figure 1

"Limit" reminds us of the experience of the Israelites, as they traveled through the wilderness. Most of the adult Israelites who came out from Egypt did not enter the Promised Land except for Caleb and Joshua. The children of Israel "approached" the Promised Land; generally speaking, all of them reached the border. But none of them would have made it were it not for God's limitless love and grace. Even though they disobeyed Him so many times, God still kept His covenant with the Israelites.

And thus, the limitless love of God is demonstrated in many other ways: in the parable of the lost sheep, the parable of the prodigal son, and even in the way God deals with His people today. Jeremiah 31: 3 (NIV) says: "I have loved you with an everlasting love." 2 Chronicles 16: 34 (NIV) says: "His love endures forever."

Let's define continuity. The idea of continuity rules out breaks, jumps, or holes by demanding that the behavior of the function near a point be consistent with its behavior at the point.

Definition: The function ***f*** is said to be continuous from ***a*** if and only if the following three conditions are satisfied:

i) $f(a)\ \text{exists};$

ii) $\lim\limits_{x \to a} f(x)\ \text{exists};$

iii) $\lim\limits_{x \to a} f(x) = f(a)$

If one or more of these three conditions fail to hold at ***a***, the function is said to be discontinuous at ***a***.

Illustration: A wholesaler who sells a product by the kilogram (or fraction of a kilogram) charges \$2 per kilogram if 10 kg or less is ordered. If more than 10 kg is ordered, the wholesaler charges \$20 plus \$1.40 for each kilogram in excess of 10 kg. Therefore, if x kilograms of the product is purchased at a total cost of $C(x)$ dollars, then $C(x) = 2x \text{ if } 0 \le x \le 10$ and $C(x) = 20 + 1.4(x - 10) \text{ if } 10 < x$; that is,

$$C(x) = \begin{cases} 2x & \text{if } 0 \le x \le 10 \\ 1.4x + 6 & \text{if } 10 < x \end{cases}$$

Solution: i) $C(10) = 2(10) = 20$

ii) $\lim_{x \to 10^-} c(x) = \lim_{x \to 10^-} 2x = 20$ $\quad \lim_{x \to 10^+} c(x) = \lim_{x \to 10^+} (1.4x + 6) = 20$

iii) $\lim_{x \to 10} c(x) = 20 = c(x)$

Therefore C is continuous at 10.

We can now use this definition to represent our faith in God.

i) If $f(a)$ is our faith in God, our faith in God exists

ii) $\lim_{x \to a} f(x) = \lim_{ourselves \to God} trials\ in\ our\ life = our\ faith\ in\ God$

iii) $\lim_{ourselves \to God} f(x) = our\ faith\ in\ God = f(a)$

If any of the above definitions does not exist, the function is said to be discontinuous. One of the examples showing that his faith in God was present was Moses when he led the Israelites out in the desert. Moses and the people were in the desert, but what was he going to do with them? They had to be fed and feed was what he did. Moses needed to have around 1500 tons of food each day. To bring that much food each day, two freight trains, each around a kilometer long would be needed. We all know they were out in the desert, so they would need firewood to cook the food. This would take 4000 tons of wood and a few more freight trains, each a kilometer long, just for one day. And they were forty years in the desert.

They would have to have water. If they only had to have enough to drink and wash a few dishes, it would take around 11,000,000 gallons each day, and a freight train with tank cars, around 3 kilometers long, just to bring water.

Then another thing: they had to cross the Red Sea. Now, if they went on a narrow path, double file, the line would be around 1200 km long and would take 35 days and nights to get through. So, there had to be space in the Red Sea, around 5 km wide so that they could walk 500 abreast to get over in one night.

But then, there was another problem. Each time they camped at the end of the day, they needed a big space, a total of around 2000 square kilometers long.

Do you think Moses sat down to figure all this out before he left Egypt? Moses believed in God, and that God would take care of everything for him.

The following persons also demonstrated faith in God: Noah, when God asked him to build the ark, when they hadn't experienced flood or rain before that time; Abraham, when God asked him to leave his family and go to another place and also when God asked him to offer his only son Isaac.

To maintain a living and growing relation with God we need to have continuous communication with Him through prayer, meditation, and reading of His Word. We must have faith or trust in Him. According to Psalms 32: 10, "The Lord's unfailing love surrounds the man who trusts in Him".

Great is our Lord and mighty in power; his understanding has no limit. Ps. 147:5(NIV)

Derivative and Unchanging God

Derivative is a mathematical tool that is used to study rate at which physical quantities change. It is one of the two central concepts of calculus, and it has a variety of applications, including curve sketching, the optimization of functions, and the analysis of rates of change.

A typical problem to which calculus can be applied is profit optimization. For example: a manufacturer's monthly profit from the sale of radios was $P(x) = 400(15 - x)(x - 2)$ dollars when the radios were sold for x dollars a piece. The graph of this profit function, which is shown in Figure 2, suggests that there is an optimal selling price x at which the manufacturer's profit will be greatest. In geometric terms, the optimal price is the x coordinate of the peak of the graph.

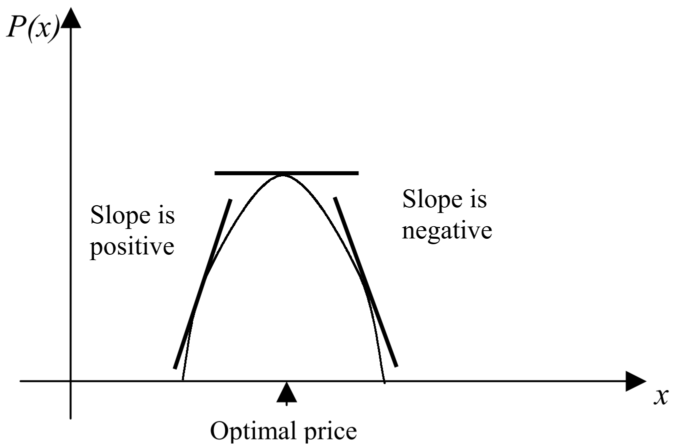


Figure 2:
The profit function $P(x) = 400(15 - x)(x - 2)$

In this example, the peak can be characterized in terms of lines that are tangent to the graph. In particular, the peak is the only point on the graph at which the tangent line is horizontal, that is, at which the slope is zero. To the left of the peak, the slope of the tangent is positive. To the right of the peak the slope is negative.

Calculus is also one of the techniques used to find the rate of change function. The rate of change of a linear function with respect to its independent variables is equal to the steepness or slope of its straight-line graph. Besides, this steepness or rate of change is constant.

We can solve optimization problems and compute rates of change if we have a procedure for finding the slope of the tangent to a curve at a given point.

We begin with a function f, and on its graph we choose a point $(x, f(x))$. Refer to Figure 3. We choose a small number $h \neq 0$ and on the graph mark the point $(x + h, f(x + h))$. Now we draw the secant line that passes through these two points. The situation is pictured in Figure 4, first with $h > 0$ and then with $h < 0$.

As h tends to zero from the right (Figure 4), the secant line tends to the limiting position indicated by the dashed line, and tends to the same limiting position as h tends to zero from the left. The line at this limiting position is what we call " the tangent to the graph at the point $(x, f(x))$.

Since the approximating secant lines have slopes

$$(*) \qquad \frac{f(x+h)-f(x)}{h},$$

we can expect the tangent line, the limiting position of these secants, to have slope

(**) $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

While (*) measures the steepness of the line that passes through the points (*x. f(x)*) and (*x +h, f(x +h)*), (**) measures the steepness of the graph at (*x, f(x)*) and is called the "slope of the graph."

We can now define differentiation.

A function ***f*** is said to be ***differentiable*** at ***x*** if and only if

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ exists.}$$

If this limit exists, it is called the ***derivative of f at x***, and is denoted by ***f'(x).***

Notations: $f'(x) = \frac{d}{dx}[f(x)]$

$$\text{Let } y = f(x),\ f'(x) = \frac{dy}{dx}$$

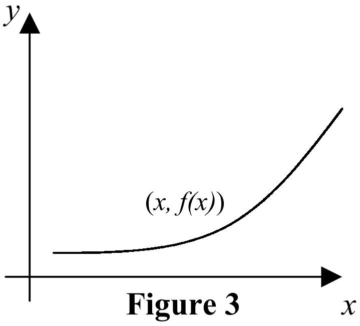


Figure 3

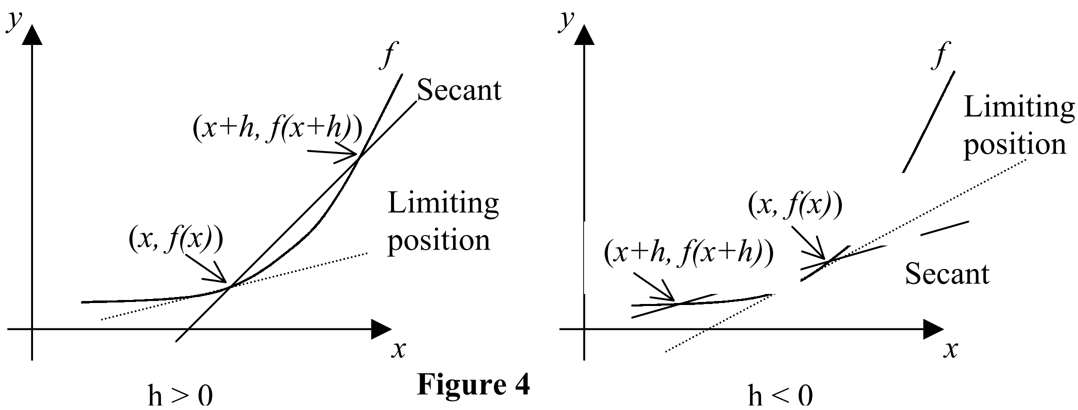


Figure 4

Note: the rate of change of a function with respect to its independent variable is equal to the steepness of its graph, which is measured by the slope of its tangent line at the point in the question. Since the slope of the tangent line is given by the derivative of the function, it follows that the rate of change is equal to the derivative.

Illustrations:

1. It is estimated that x months from now, the population of Mission College SDA church will be $P(x) = x^2 + 20x + 800$. At what rate will the population be changing with respect to time 15 months from now?

Solution:

The rate of change of the population with respect to time is the derivative of the population function. That is

Rate of change = $P'(x) = 2x + 20$

The rate of change of the population 15 months from now will be

$P'(15) = 2(15) + 20 = 50$ people per month

2.

Distance (S)	Love (L)
Velocity v= dS/dt	God G = dL/dt = 0

Table 2

Since the derivative of the constant is zero and we know that God's love is constant and it does not change, so the rate of change of God's love is zero. 1 John 4:16(NIV) says: God is love. Hebrews 13: 8 says: "Jesus Christ (God) is the same yesterday, today and forever." Hence God's love is the same yesterday, today and forever. " There was enough, and more than enough. In love there is no nice calculation of less and more. God is like that"(Barclay: p. 118)

Concavity and points of Inflection:

The graph is concave up on an open interval where the slope increases and concave down on an open interval where the slope decreases. Points that join arcs of opposite concavity are called points of inflection. The graph in Figure 5 has three of them: $(c_1, f(c_1))$, $(c_2, f(c_2))$, $(c_3, f(c_3))$. In our daily lives we also have ups and downs and we have points of inflection, where we need to make decisions. According to Joshua 24: 15(NIV), "Choose for yourselves this day whom you will serve." We cannot serve two masters; one master brings us up and the other brings us down.

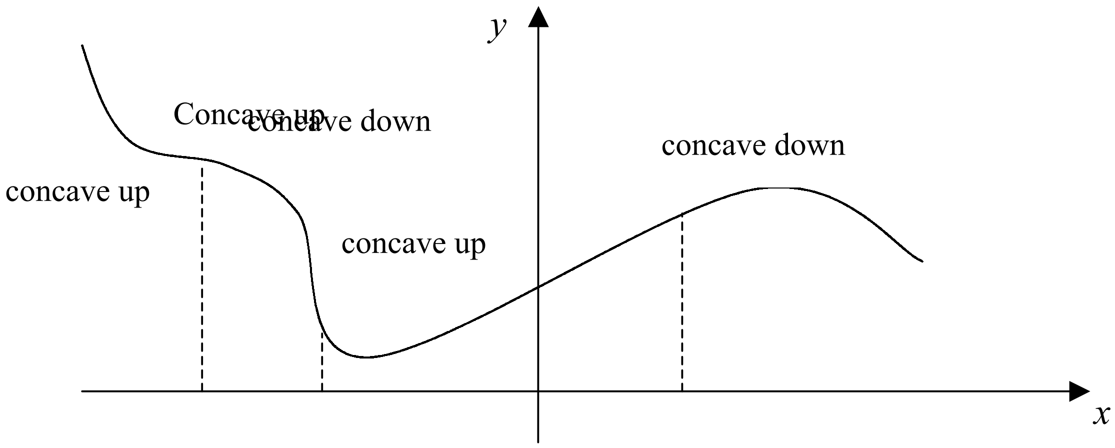


Figure 5

Integral and the Orderly God

In many problems, the derivative of a function is known and the goal is to find the function itself. For example, a sociologist who knows the rate at which the population is growing may wish to use this information to predict future population levels; a physicist who knows the speed of a moving body may wish to calculate the future position of the body; an economist who knows the rate of inflation may wish to estimate future prices. God knows the rate of progress in His people and He knows when His work will be completed, so He can reliably predict His second coming, predict the close of probation, etc.

The process of obtaining a function from its derivative is called antidifferentiation or integration.

Let's define Antiderivative:

A function ***F***(***x***) for which

$$F'(x) = f(x)$$

for every ***x*** in the domain of f is said to be an antiderivative (or indefinite integral) of ***f***.

The Antiderivatives of a Function:

If ***F*** and ***G*** are antiderivatives of f, then there is a constant ***C*** such that

$$G(x) = F(x) + C$$

Integral Notation: It is customary to write

$$\int f(x)dx = F(x) + C$$

to express the fact that every antiderivative (integral) of ***f***(x) is of the form ***F(x)*** + ***C***. For example, we can express the fact that every antiderivative of $3x^2$ is of the form $x^3 + C$ by writing

$$\int 3x^2\,dx = x^3 + C$$

The symbol $\int$ is called an integral sign and indicates that you are to find the most general form of antiderivative or integral of the function following it.

To apply integration we need to follow specific rules, for example,

1. The power rule of Integrals; $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, for n$\neq$-1.
2. The Integral of $\frac{1}{x}$; $\int \frac{1}{x}dx = \ln|x| + C$
3. The Integral of e^x; $\int e^x\,dx = e^x + C$
4. The Constant Multiple Rule for Integrals;
$$\int c\,f(x)dx = c\int f(x)dx$$
5. The Sum Rule for Integrals;
$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

There are more than 120 rules or properties that can be used in Integration. There is a specific rule that we apply for each kind of function. When God created this world, everything He made had a specific role to play. For example, the sun would give us light; air was for breathing; water was for drinking and cleansing; trees, animals, and human beings were to live symbiotically; gravity would keep everything from bouncing. And He put everything in order. God is a God of order. 1 Corinthians 14:33 (NIV) says, " God is not a God of disorder but of peace."

Suppose that the known the rate $f(x) = \frac{dF}{dx}$ at which a certain quantity ***F*** is changing and we wish to find the amount by which the quantity ***F*** will change between ***x = a*** and ***x = b***. We would first find F by antidifferentiation and then compute the difference. The numerical result of such a computation is called **definite integral** of the function ***f*** and is denoted by the symbol

$$\int_a^b f(x) = F(b) - F(a).$$

Applications of Integrals:

1. Calculate the population of the town ***x*** months from now.
2. Total cost of producing ***x*** units.
3. Total consumption over the next ***t*** years.
4. Area
5. Volume up to three dimension

Illustration: Area in Polar Coordinate

$$A = \int_a^b \frac{1}{2} r^2 d\theta$$

The hardest part in applying the Area formula is determining the limits (boundary) of integration. This can be done as follows:

1. Sketch the region R whose area is to be determined.

   Genesis 2:7 (NIV) says: " God formed man of dust from the ground."

2. Draw an arbitrary "radial line" from the pole to the boundary curve

   $$r = f(\theta)$$

   Genesis 1:27 (NIV) says: " God created man in His own image"

3. Ask, "Over what interval of values must θ vary in order for the radial line to sweep out the region R?"
4. Our answer in Step 3 will determine the lower and upper limits of integration.

   Psalms 8:5 (NIV) says: " Yet you have made him a little lower than God."

Example:

Find the area of the region in the first quadrant within the cardioid

$r = 1 - \cos\theta$

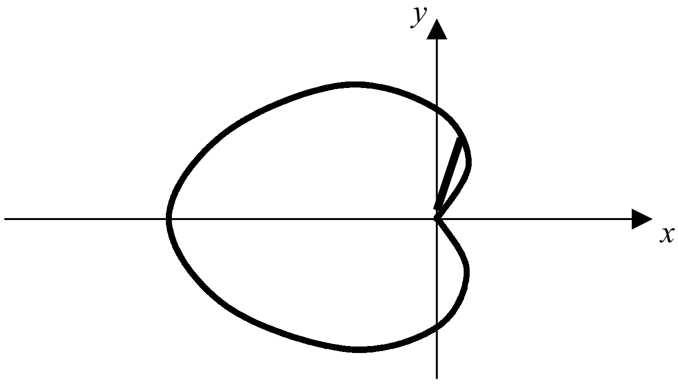
Solution:

The region and a typical radial line are shown in Figure 5. For the radial line to sweep out the region, θ must vary from 0 to $\frac{\pi}{2}$.

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos\theta)^2 d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \cos^2\theta) d\theta$$

$$A = \frac{3}{8}\pi - 1$$

To find the area of the region we need to find the point where the radial line (point of reference). In order for us to find direction in our life we need to have a point of reference, which is God.



The shaded region is swept out by the radial line as θ varies from 0 to $\frac{\pi}{2}$.

Figure 5

Conclusion

From the above examples, we have shown some ways by which a mathematics teacher can use concepts in calculus to help students understand more about God. Calculus, or any subject, should not be taught in isolation but should be related to life and faith. True education should prepare a student not only for this life but also for the life to come. "True education means more than a preparation for the life that now is. It has to do with the whole being, and with the whole period of existence possible to the man. It is the harmonious development of the physical, the mental, and the spiritual powers. It prepares the student for the joy of the service in this world and for the higher joy of wider service in the world to come." (White, Counsels on Education) It should introduce the student to God and get him ready to enjoy His presence for eternity.

" Mathematics is a revelation of the thought life of God. It shows Him to be a God of system, order, and accuracy. He can be depended upon. His logic is certain. By thinking in mathematical terms, therefore, we are actually thinking God's thoughts after Him." (SDA, p.6)

References

1. Anton, Howard. Calculus A New Horizon. 6th ed. John Wiley, 1999.

2. Barclay, William. The Daily Study Bible Series: the Gospel of Luke. Westminster John Knox Press Louisville, Kentucky.

3. Hoffmann, Laurence and Bradley, Gerald. Brief Calculus with applications. 5th ed. McGraw Hill, 1993.

4. Hughes-Hallett, Gleason, McCallum, et al. Calculus, Single and Multiplevariable, 2nd ed. John Willey, 1998

5. Holy Bible (NIV)

6. Salas, Hille, and Etegen. Calculus One and Several Variable. 8th ed. John Wiley, 1999.

7. SDA Secondary Curriculum (Mathematics).

8. Thompson, Avery J. "God in Nature: Revelation of the Divine Mathematician." Christ in the Classroom. Vol. 17.

9. White, Ellen G. Counsels on Education.

10. White, Ellen G. Education.